**Discrete age-structured SEIR epidemic model with applications to measles vaccination strategies**

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A Research Proposal for the Biomathematics Honours Project

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August 2022

**Introduction**

The age distribution is the most significant element that makes contributions to the heterogeneity of populations, which has a substantial impact on the timing and effects of the transmission and spread of infectious diseases [1]. Most crucially, there is a considerable degree of non-uniformity in transmission rates due to the patterns and frequency of individual encounters, which can range dramatically between age groups [2]. Individuals of different ages can also have different levels of immunity against infectious diseases. Age-specific mortality and recovery rates from an infection may be impacted by these variations [3].

Measles is a Vaccine-Preventable Disease (VPD) [4]. A vaccine is the most effective way to prevent infectious diseases [5]. The measles vaccine is commonly administered to infants as part of the Measles-Mumps-Rubella (MMR) vaccination and requires two doses. However, the measles vaccine is normally administered to children at 6 months old age, and the second dose is administered at 12 months old age [6].

For this study, we are interested to comprehend how an infectious disease spreads in a host population with an age structure, we examine an SEIR epidemic model with discrete age groups with the application of measles vaccination strategies. Each age group has different vaccine coverage and efficacy. The efficacy of the first dose of the measles vaccine is scheduled for 6 months old, whilst the efficacy of the second dose of the measles vaccine is scheduled for 12 months old age. Vaccine coverage of first and second doses is meant to reduce the measles incidence rate.

To conduct this study, we will build an SEIR epidemiology model is then to analyse the measles data in South Africa and evaluate the effectiveness of several vaccination strategies for the control of measles epidemics. The model will be fitted to some real data. Data simulated by our model can be used to answer our desired research question. We should be able to use our model to distinguish between scenarios of efficacy and vaccine coverage.

**Research question**

The proposed research question is investigating the Measles SEIR epidemic model with discrete age groups to understand the transmission dynamics of an infectious disease in a host population with an age structure. The main goal to answer the proposed research question is to explore the transmission dynamics of an infectious disease in a host population with an age structure. We will adapt the SEIR epidemic model with a discrete age structure to study the vaccination strategies for measles.

**Research objectives**

* The epidemic model will be used to analyse the measles data in South Africa and evaluate the effectiveness of several vaccination strategies for the control of measles epidemics in South Africa.
* We will calibrate an age-group model for measles vaccination using published data by The National Institute for Communicable Diseases on measles incidence from South Africa, and compare the effects of two different measles vaccination strategies proposed by the NICD together with the World Health Organisation.

**Study benefits**

The research of the discrete age structure epidemic model with the application of the measles vaccination strategy will bring outcomes on the effectiveness of vaccination strategies and measles eradication targets for the upcoming future predictions. The modified measles vaccination model with two age groups that incorporated the current measles vaccination programs will assist in the analysis of the immune profile for the population and in each age group to establish the baseline and make future projections.

The model will subsequently reveal the effect of increasing vaccine coverage to be greater or equal to 95% with two doses administered for each person. The study will also show the impact of increasing the efficacy of two doses in reducing the effectiveness of measles incidences at a moderated vaccine coverage rate.

**Mathematical Model**

**An application to vaccination strategies for measles**

Measles is a disease that can be prevented with a vaccine [7]. The measles vaccine, given as part of the measles, mumps, and rubella (MMR) vaccine, usually requires two doses. The first dose of the measles vaccine is commonly given to infants at 6 months of age and the second dose is administered at 12 months of age [8]. The efficacy of two doses of the measles vaccine range from 93% to 99% [9]. In South Africa, vaccine coverage requires a maximum of 95% or higher to be sustained with both doses administered per person [10].

In this section, we develop a two-dose vaccination version with two age corporations to observe the vaccination strategies for measles epidemics.

**Measles vaccination model**

Measles can be prevented with the MMR vaccine. The WHO recommends that children receive the MMR vaccine twice. The first dose is 6 months old and the second dose is 12 months old [6]. One dose of MMR vaccine is 93% effective against measles while two doses of MMR vaccine are 97% effective against measles [10]. In South Africa, vaccine coverage for children at 12 months old age averaged 71.1%, while the second dose averaged 68.8% between the year 2012 to 2017. The coverage of the second dose increased to 76.4% in 2018 [9].

We subdivide the host population into two age groups, considering age-specific differences in vaccination schedules, mortality, and contact patterns.

Figure 1: Transfer diagram for a vaccination model with two age group

The model structure is shown in the transmission diagram in Figure 1. Two doses of measles vaccine were incorporated: MMR1 for age group 1 (6 months), and MMR2 for age group 2 (12 months). The model is described by the following system of differential equations.

Differential equations for age group 1:

Differential equations for age group 2:

The model parameters are shown in Table 1 along with their description and parameters. Specifically, and are the vaccination rates of MMR1 and MMR2, respectively, and are the efficacy of MMR1 and MMR2, respectively, and and are the effective coverage of MMR1 and MMR2, respectively.

To incorporate vaccination, assume a proportion, , of 6-month-old into the population are vaccinated (and thus immune to infection). Vaccinated people avoid susceptible classes and go directly to recovered classes, while unvaccinated people go to susceptible classes as before. If is the proportion vaccinated, then is the proportion left unvaccinated.

The transmission coefficient between and is decomposed into two factors where is the probability of transmission for an average contact between a susceptible individual in age group with infected individual, and is the mean number of contacts between people in age group and people in age group . Note that and are not the same and the contact matrix may not be symmetric because of different ages.

Influx susceptible individuals are recruited by the rate of . Exposed individuals move to the infectious class at a rate of an age group of . Infectious individuals move to the recovered compartment at a rate of an age group of . Individuals are aging at a rate αk. Natural fatality rate of an age group is represented by dk, while case fatality of an age group is represented by a rate of . The aging rate of age group is and individuals gain of immunity at rate

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Parameters | Values/Range | Unit | Description | Ref |
|  | 650 000 |  | Influx of susceptible | fitting |
|  | 0.2 |  | The natural mortality rate of age group k | fitting |
|  | 0.3 |  | Aging rate of age group k | fitting |
|  | 0.25 |  | Recovery rate of age group k | fitting |
|  | 0.72 |  | Exposed rate of age group | [13] |
|  | 0.2 |  | Case mortality rate of age group k | fitting |
|  | 0.717 |  | Vaccination coverage of Measles vaccine | [10] |
|  | 0.764 |  | Vaccination coverage of Measles vaccine | [10] |
|  | 0.93 |  | Efficacy of MMR1 | [9] |
|  | 0.95 |  | Efficacy of MMR2 | [9] |
|  | 0.1679 |  | Probability of transmission per contact for age group 1 | fitting |
|  | 0.5154 |  | Probability of transmission per contact for age group 2 | fitting |
|  | 0.4 |  | Gain of immunity of age group k | fitting |
|  | 5 |  | Average number of contacts from age group j to age group k | fitting |

Table 1: Parameters and their estimated values for model

**Parameters estimation and model calibration**

As indicated in Table 1, values of some parameters and initial values of state variables in model are estimated directly from published data. Other parameter values, especially those of the probability of transmission per contact and the recovery rate from measles for each age group, are estimated by fitting the model outcomes to measles data using the nonlinear least squares method [11]. The measles data used for model fitting include the reported annual incidence and age specific incidence of measles in India from 2015 to 2020 [12]. The values of measles case mortality ratio are and . By the end of 2020, the values of , are the actual vaccination rates published by NICD [9].

**Immune profile analysis**

In our model, we will generate the measles immune profile analysis for total population and for different age group. The endemic level of measles vaccination strategies in South Africa, namely, a single-dose vaccine at 6 months old (age group 1) and the second dose vaccine at 12 months old (age group 1) during years 2015-2020. In South Africa, vaccination coverage of children under 1 year averaged 71.1% , whilst measles second dose vaccination coverage is 76.4% [9]. The efficacy of two doses of measles vaccine ranges from to 93-99%. We therefore assume that the efficacy of the first dose is 93% and for the second dose is 95% [10].

**Effect of increasing measles and improving vaccination coverage**

Vaccination is critical to sustaining and increasing vaccination coverage rates and preventing outbreaks of measles vaccine preventable disease. The strong enforcement may help promote higher rates of vaccination coverage along with complementary actions such as monitoring VPD cases. The vaccination coverages of single-dose (MMR 1) and the second dose (MMR 2) should be both increased to 95% [10]. In South Africa, the efficacy of MMR 1 can reaches 99% when administered to children 6 months old.

**Code design**

The discrete age-structured SEIR epidemic model with applications to measles vaccination strategies we will be implementing computational language. The programming language that will be used is **R** specifically R-Studio to code our epidemic model. We used *tidyverse* package.

We create a function to calculate the rate of change in each state variable. This function solves our Ordinary Differential Equations (ODE’s), this function takes parameters of the model system. This function that will update the system at each time step. The change in state variables is calculated and returned in our function, we calculated this by using differential equations from our model.

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